The Maxwell wheel investigated with MBL

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Abstract: The Maxwell wheel motion is experimentally investigated by interfacing the system to a PC with a force sensor and a position sensor. Mechanical energy transformations and the impulse-momentum theorem during the “collision” at the end of the string are studied.

Introduction

The Maxwell wheel is a device traditionally used in the introductory physics lab in order to investigate the moment of inertia of a disc. It is made of a disc of radius *R* and an axis of radius *r* (with $r << R$). Two (equal length) wires wound around the axis have one end attached to the axis and the other end attached to a fixed frame (figure 1)[1].

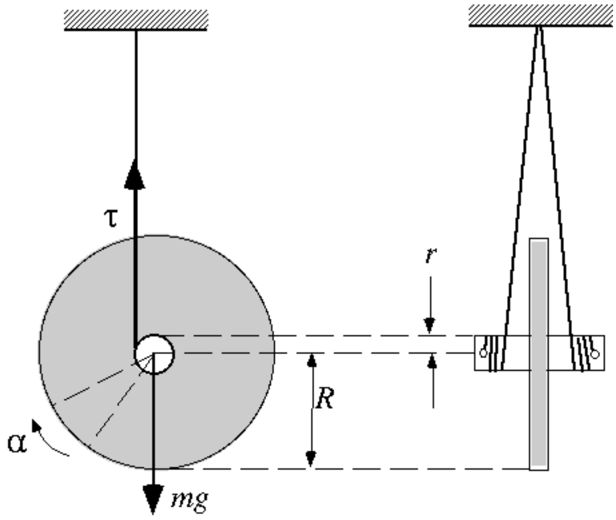

Figure 1

Which are the most apparent features of the Maxwell wheel motion ? At first glance we see that the velocity and therefore the acceleration of the "fall" are smaller than those of a free falling body. Secondly we see that it rotates. More precisely: to get down it must rotate, and when it reaches the wire-end in order to maintain the rotation it must climb up. Finally we notice that in the lowest position it does stretch the wire.

The aim of this paper is to illustrate the didactic advantages of a *kinematic* and *dynamic* study of the Maxwell wheel motion by an on line apparatus. We will stress the methodological aspects characterising our approach: extensive use of graphic representations, design and control of a first approximation physical model, gradual refinement of the model guided by the experimental results.

The experimental set up

By interfacing a Maxwell wheel to a Personal Computer through a position sensor (sonar) and a force sensor (figure 2) the system evolution in time may be observed and recorded both kinematically (from *position* measurements *velocity* and *acceleration* are calculated) and dynamically (the force transducer measures the wire *tension*).

[1] If we attach the two wires to a single point, they will spirally wound around the axis avoiding changes of the effective radius *r*.

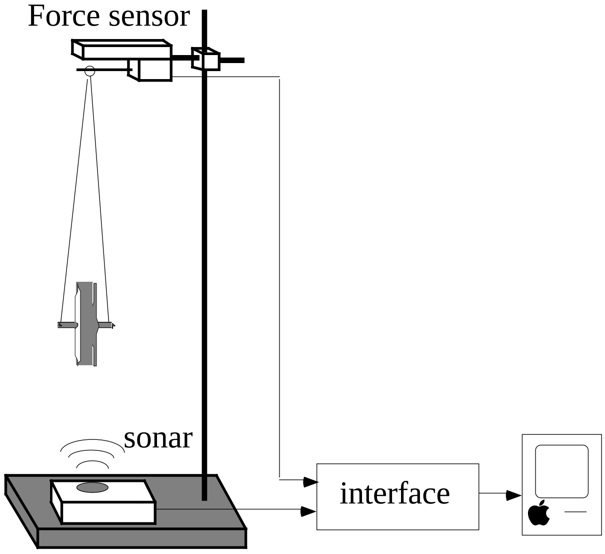

Figure 2

We used two types of low-cost interfaces specially suitable for didactic applications (ULI II[2] and PASCO[3] 500), both bundled with software designed for easy data acquisition and handling. The interface communicates with the PC through the slow RS232 serial port (COM in Windows and AppleTalk in Macintosh), but an internal buffer memory allows to collect data at high rate.

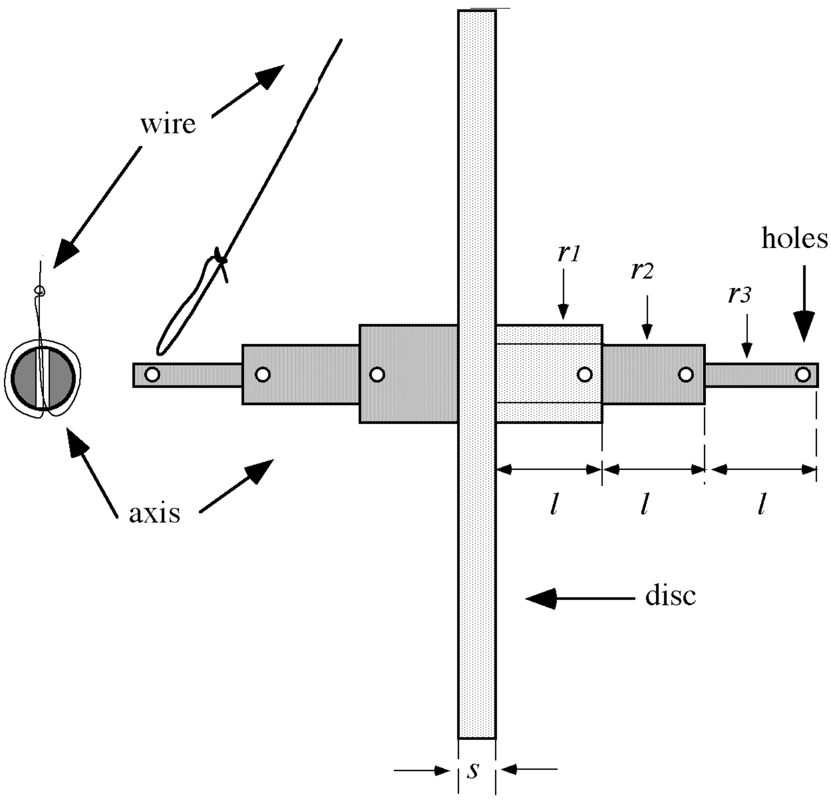

figure 3

[2] Produced by Vernier Software, Portland OR, http://www.vernier.com

[3] Produced by PASCO Scientific,Roseville, CA, http://www.pasco.com

Our Maxwell wheel (shown in figure 3) is obtained from a thin metal disc of radius R with a central hole hosting a axis that has three sections with different diameters (4.0 mm, 6.3 mm and 8.0 mm) to give three values for the ratio R/r. At both ends of each section thin transverse holes (1 mm) are drilled as posts for attaching the wires.

An example of motion acquisition

Using a brass wheel of radius R=47.5 mm, thickness 3 mm, and mass m=227 g, with axis radius r=2 mm, we obtained the plots shown[4] in figure 4.

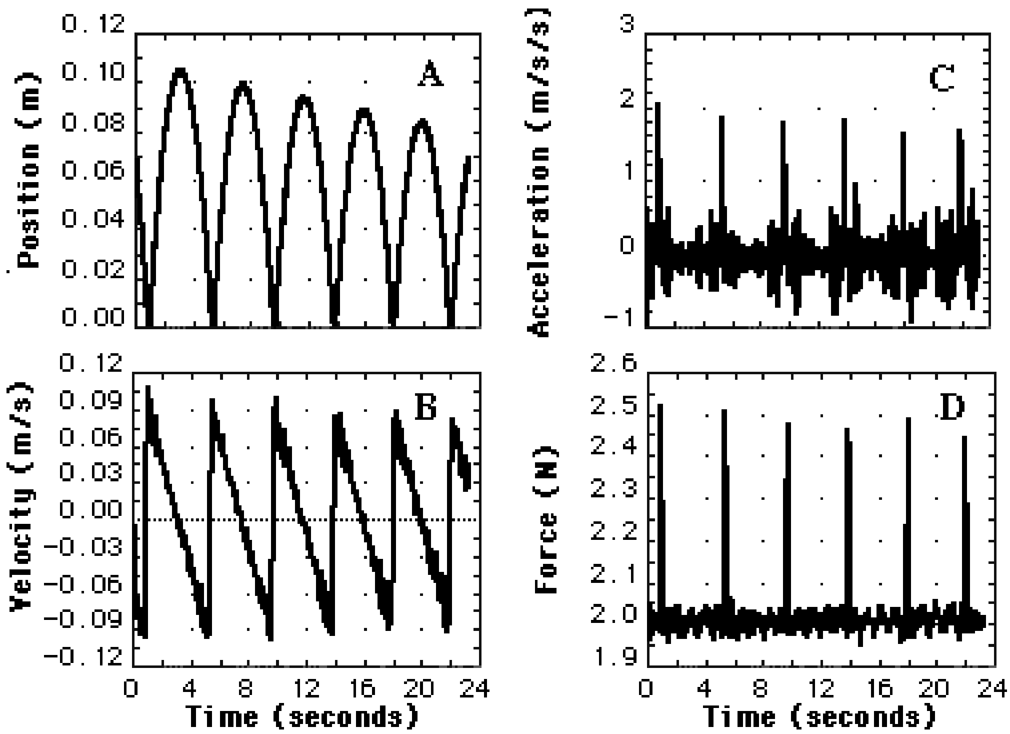

Figure 4

We note that the position vs. time plot (A) reminds that of a *bouncing ball.* The velocity plot (B) shows *discontinuities* corresponding to the "bounces", where the velocity *suddenly changes sign* . The plot (C) shows that the acceleration is *negative* and *small*, during climb and fall, with *positive peaks* at the bottom positions. The force plot (D) shows that also the *wire tension* has corresponding *peaks* .

A physical description of the upward and downward motion

The simplest model is based on the energy considerations. If we let the wheel free from an height h, and the potential energy $U(x)$ is referred to the equilibrium position (U=0 at x=0), the system total energy E at start ($x=h$, $v=0$) is $E=mgh$. During the fall the *potential* energy transforms into *translational kinetic* energy $E_t=mv^2/2$ and *rotational kinetic* energy $E_r=I\omega^2/2$. Assuming negligible dissipation we may write therefore: $E=U(x)+E_t(x)+E_r(x)$, or $mgh = mgx + \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$, where m

[4] The noise on acceleration data is due to the effect of the double derivative of position data**.**

is the Maxwell wheel mass, I the momentum of inertia and ω the angular velocity.

This energy balance does explain why the linear acceleration is less than g.

To experimentally test the model we should write the acceleration as a function of directly measurable quantities. To do this we may exploit the fact that the translational and the rotational motions are not independent.

Using an upward vertical axis with the origin in the equilibrium position, the rotation angle $\alpha(t)$ is related to the displacement $x(t)$ by: $x(t) = r\,\alpha(t)$ (where r is the axis radius), while the angular velocity is related to the linear velocity by:

$$v(t)=r\,\omega(t). \quad (1)$$

We may therefore write all the quantities as functions of the *position* x and of the *velocity* v so that the energy balance becomes:

$$mg(h-x)=\tfrac{1}{2}mv^2+\tfrac{1}{2}I\omega^2=\tfrac{1}{2}m\left(1+\frac{I}{mr^2}\right)v^2=\tfrac{1}{2}(mk)v^2$$

showing that the system behaves like a point with inertial mass $m\,k = m(1+I/mr^2)$

Solving with respect to the velocity, we get a relation similar to that for the velocity of a free body falling from height h:

$$v(x)=\sqrt{\frac{2g(h-x)}{1+I/(mr^2)}}=\sqrt{2\frac{g}{k}(h-x)}=\sqrt{2\,a\,(h-x)}\ ,$$

showing that the acceleration of the center of mass is[5]

$$a=g/k=g/(1+I/mr^2) \quad (2)$$

For $r<<R$, the momentum of inertia I of the wheel does well approximate that of the disc ($I\approx mR^2/2$), and therefore the predicted acceleration is

$$a\approx g/\left(1+R^2/2r^2\right)\approx g(2r^2/R^2) \quad (3)$$

A check of the predictions derived from the model

The momentum of inertia of an homogeneous cylinder of density ρ, radius R and length s is $I=\pi R^4 s\rho/2$. For our Maxwell wheel, made of 4 cylinders, we get :

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[5] We may reach the same result by equating the torque T, applied by the external force, to the derivative of the angular momentum dL/dt. If we calculate the momenta with respect to the axis of symmetry of the wheel, the driving torque is $T=r\tau$, where τ is the sum of the vertical components of the wire tensions (the gravitational force mg, applied to the center of mass, has zero momentum). The angular momentum is $L=I\omega$, and therefore $r\tau=Id\omega/dt=I(a/r)$, that gives the tension $\tau=aI/r^2$. Using the second Newton's law for the net force $mg-\tau$ applied to the wheel we get the equation $ma=mg-\tau=mg-aI/r^2$. Solving with respect to a we obtain : $a = g\,[m\,/\,(I/r^2+m)]=g/k$

$$I = (\pi\rho/2)[R^4 s + (r_1^4 + r_2^4 + r_3^4)2\ell] = (\pi\rho R^4 s/2)\left[1+\left\{\frac{r_1^4+r_2^4+r_3^4}{R^4}\,\frac{2\ell}{s}\right\}\right]$$

With R=47.5 mm, s=3.0 mm, r_1=4.0mm, r_2=3.2mm, r_3=2.0mm, and l=20mm, the term within curl brackets is negligible (1.9×10^{-3}), and we may therefore approximate I with the momentum of inertia of the disc only: $I\approx mR^2/2$, letting $k\approx(R/r)^2/2$.

With the values chosen in the experimental set up of figure 4 we obtain $k\approx283$ and a= 0.035 m/s^2,

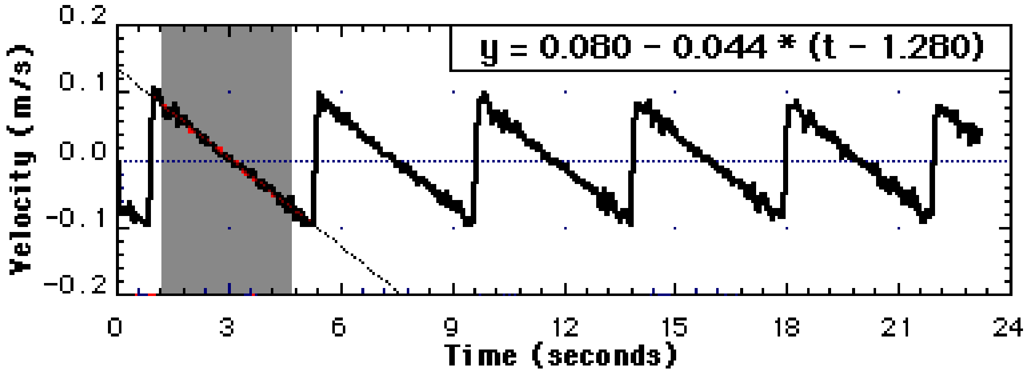

figure 5

The value a =0.044m/s^2, obtained by fitting linearly the velocity data (see figure 5), has the predicted order of magnitude, but it is in excess of more than 20%. Therefore we must conclude that in our model we neglected some important feature of the studied phenomenon. We may observe that we assumed a zero thickness of the wire, that in our case is about 0.4 mm.

The effective value of the axis radius must be increased of about half of the wire thickness (0.2 mm). This lowers the k value to 232, and the predicted value of the acceleration becomes a =0.042 m/s^2.

The finite wire thickness introduces a correction of 18% (much greater than that due to the momentum of inertia of the wheel axis ≈ 0.2%). This tells us that details at first sight negligible may turn out to be quite important.

The “static” and “dynamic” disc weight

The force sensor records the sum of the vertical components of the wire tensions, revealing, for examples, a change when, having blocked the disc with an extra-wire, we cut this wire and let the wheel fall.

In our model the force measured with the steady disc must equal the weight $\tau_o=mg$, and with the falling disc it must be $\tau_1=mg-ma$..

With an aluminum disc weighting 80 grams the static tension is τ_o=0.786 N and the predicted difference is $\Delta\tau=\tau_o-\tau_1=ma$ =(0.08 Kg)×(0.1ms^{-2})≈ 0.008 N, the value for a being obtained from the experimental data (see figure 6C).

Here the use of a larger axis radius (r=3.2 mm) to make the acceleration larger is recommended in order to make the variation detectable.

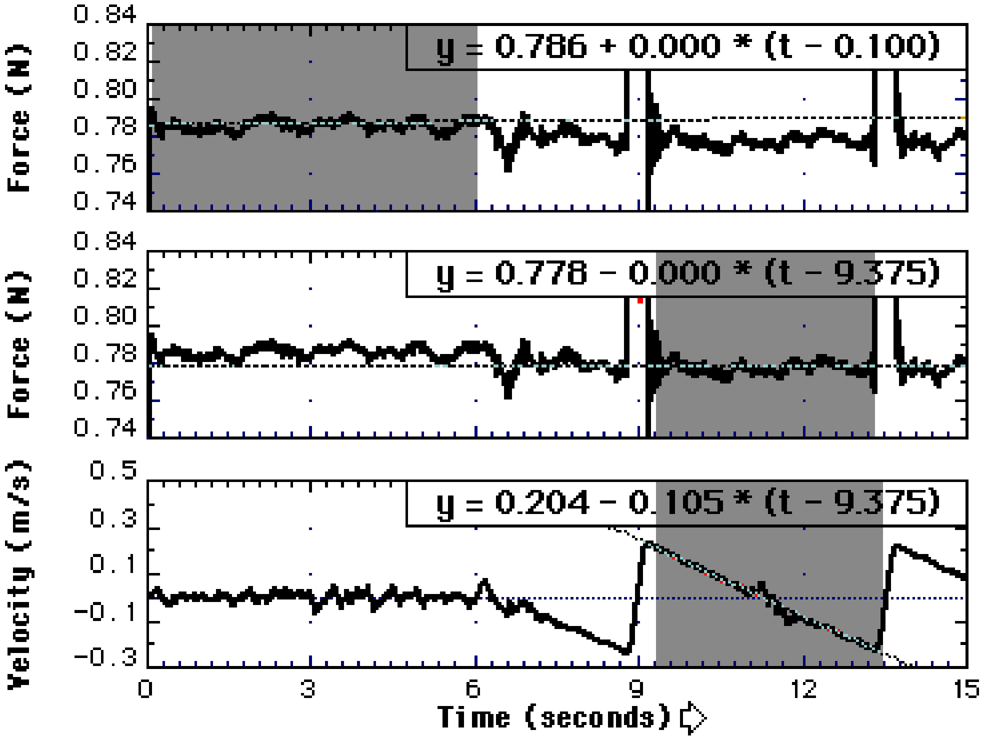

Figure 6

Recording the force *before* and *after* the wire-cut[6] we get the tension values τ_o=0.786 N and τ_1=0.778 N (see figure 6B) whose difference is consistent with the expected value.

The "collision" at the end of the string

When the Maxwell wheel reaches the end of the wire it has a momentum *mv* directed *downward*. This downward motion is stopped and reversed by the *wire stretching*, that mimics a *collision* against an "invisible wall". The collision at the wire end takes place within a short time (Δt≈0.16 seconds). During this time the translational kinetic energy is firstly converted into elastic energy, stored within the stretched wire (and the strained force sensor), and then given back again as translational kinetic energy now associated with an *upward* motion.

[6] The glitch in the plot τ(t) at the beginning of the fall is due to the scissors cutting the wire.

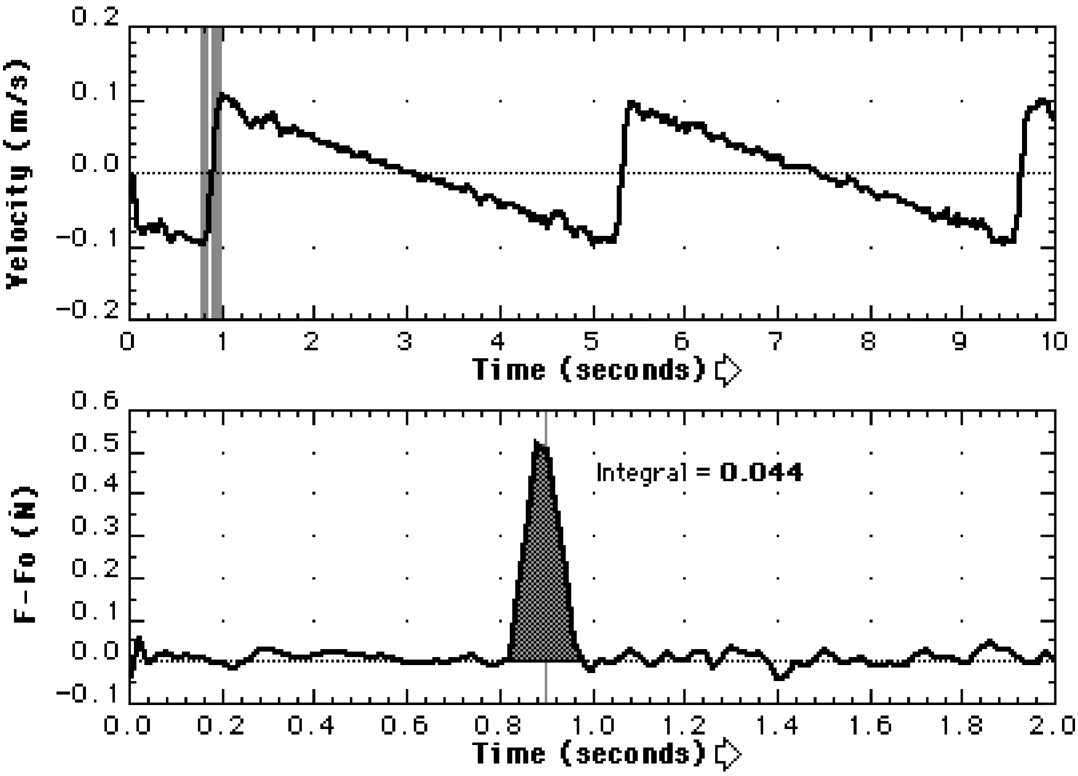

Figure 7

Newton's second law relates the momentum change to the impulsive force f_i(t): in differential form $f_i = ma = m(\mathrm{d}v/\mathrm{d}t)$, in integral form $m\Delta v = \int f_i(t)\,dt$. The impulsive force is here the *change* $\tau - \tau_o$ of the wire tension, measured by the force sensor.

In figure 7 we report on an expanded time-scale the same data shown in figure 5 in order to compare the *momentum change* $\Delta(mv)$ to the *integral of the pulse* $f_i\mathrm{d}t$.

The integral, computed by the dedicated software as the area under the curve $\tau(x) - \tau_o$ gives the value 0.044 Ns, very close to the value of the product $m(v_2 - v_1)$ = (0.227 Kg)×(0.2 m/s^2)= 0.045 Ns.

Energy vs. time

The total energy of the wheel during the upward and downward motion may be calculated from the measured values of the position x and of the velocity v of the center of mass, as:

$$E(x,v) = mgx + \tfrac{1}{2}mv^2 + \tfrac{1}{2}I\omega^2 = mgx + \tfrac{1}{2}m\left(1 + \frac{I}{mr^2}\right)v^2 = mg\left(x + v^2/2a\right) \qquad (4)$$

where we used relation (2) to express I/mr^2 in terms of g and a (calculated from experimental data as in figure 8A). The calculated values of the total energy, plotted versus time in figure 8B, show that the sum of the potential and kinetic energy during each cycle of the downward and upward motion is almost constant as expected. In this plot energy seems to disappear during the collisions. This is due to two different limitations in the way total energy has been computed. The first is that relation (4) is based on relation (1) which is not valid when x reaches values close to zero, i.e. during the collision at the wire end. In fact during the wire stretching the linear velocity changes sign (passing through zero) while the angular velocity remains approximately constant. The second is that relation (4) does not take into account the elastic energy of the stretched wire.

Keeping the two limitations in mind we find (figure 8B) that during upward and downward motion the total energy decrease can be fitted exponentially (with a time constant of about 55 s). Since the decrease appears to occur uniformly during the whole motion it seems mainly due to viscous friction.

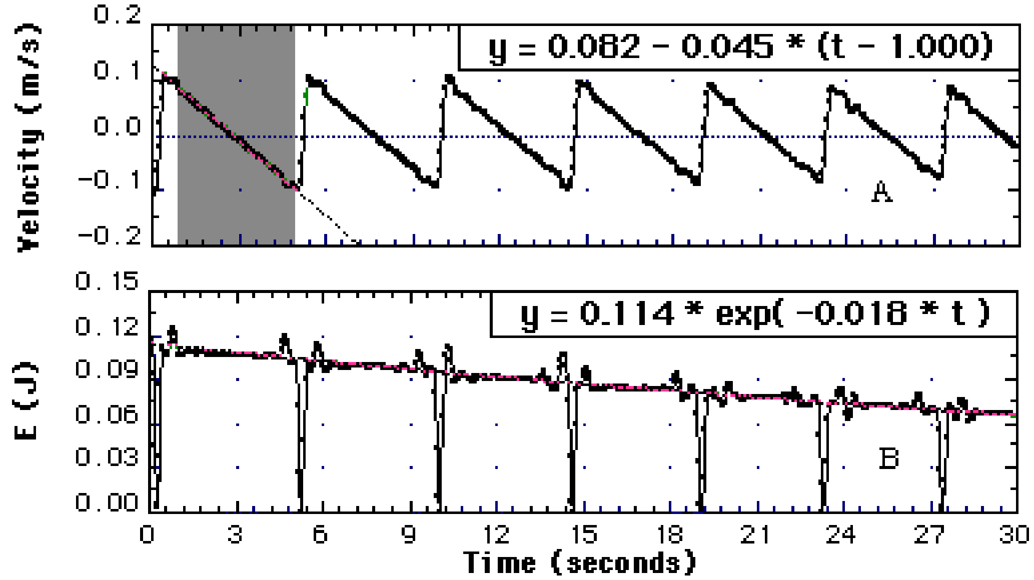

Figure 8

Suggestions for teaching

A more familiar version of the Maxwell wheel is the traditional toy called yo-yo. The experimental set up described before can be used for the experimental investigation of the behavior of a yo-yo, which may prove a good start for the study of translational and rotational motions.
A record of the experimental data obtained with a commercial yo-yo (a plastic toy with mass m=18 g and axis radius r=3.5 mm) is shown in figure 9.

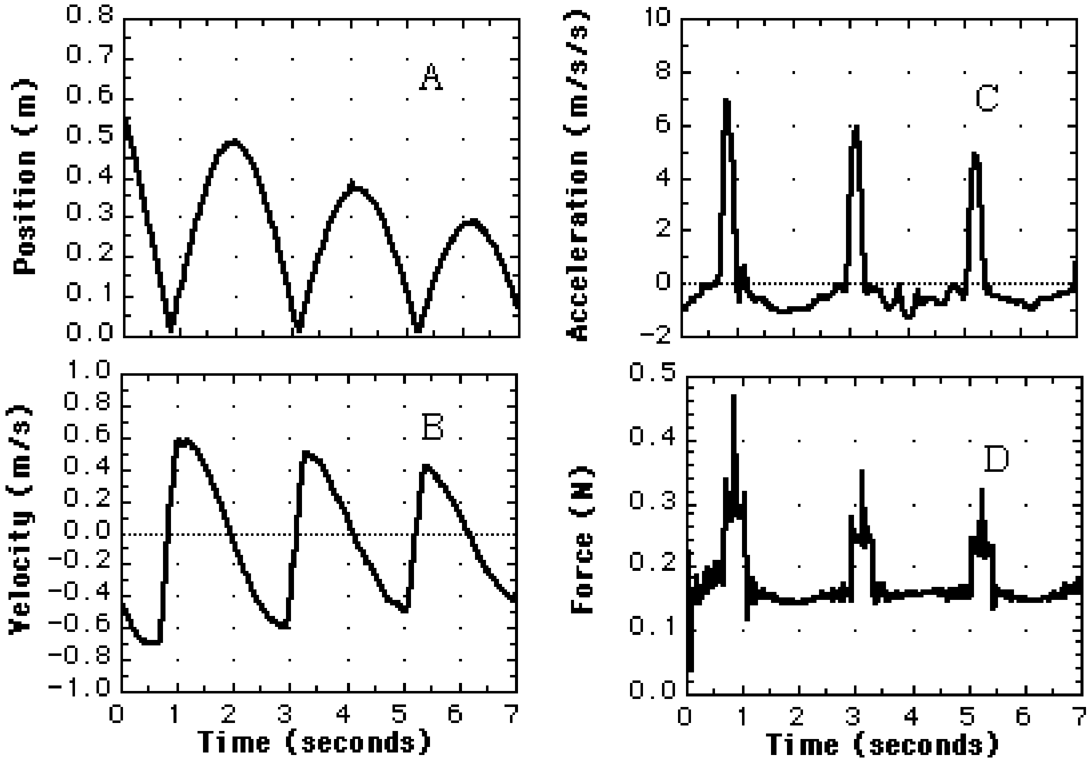

Figure 9

To compare the model predictions with the experimental results we should know r, m and I.
But for a yo-yo it is difficult to evaluate the momentum of inertia, being its shape not a geometrically simple one.
However it may be fruitful to use the plots in order to try a qualitative check of our model. During the upward and downward motion we expect a negative (downward as g) acceleration, constant, and less than g .
The behavior of the position versus time is qualitatively the expected one: the plot seems to be made of parabolic branches (figure 9A). However looking at the velocity, we see that it has not the predicted constant slope, during the upward and downward motion (figure 9B). This is apparent also in the acceleration versus time plot, that shows *modulations* even far from the instants when the yo-yo bounces back (figure 9C). We can also notice that the absolute value of the acceleration *is minimum when the wire is almost completely wound off*.
How do we explain this modulation? A possible explanation is that the effective radius of the axis does change....because the wire winds over itself and this acts like an axis with increasing diameter.
In other words when the wire is almost all wound-up on the axis (velocity ≈ 0, and the yo-yo in the topmost position) the axis *effective radius* is larger than the r value that our model assumes.
The Maxwell wheel can therefore be introduced as a refined version of the toy, intentionally designed in order to avoid the difficulties met in the previous investigation.
It may also be stimulating for the students to realize that, as hinted before, the graphs of the kinematic description of the motion of the wheel closely resemble those of the motion of a bouncing ball.
In figure 10 we report experimental data collected with a position sensor located above a ping-pong ball bouncing on the floor (the ball position is nevertheless recorded as distance from the floor, in order to allow easier comparison with the previous graphs).

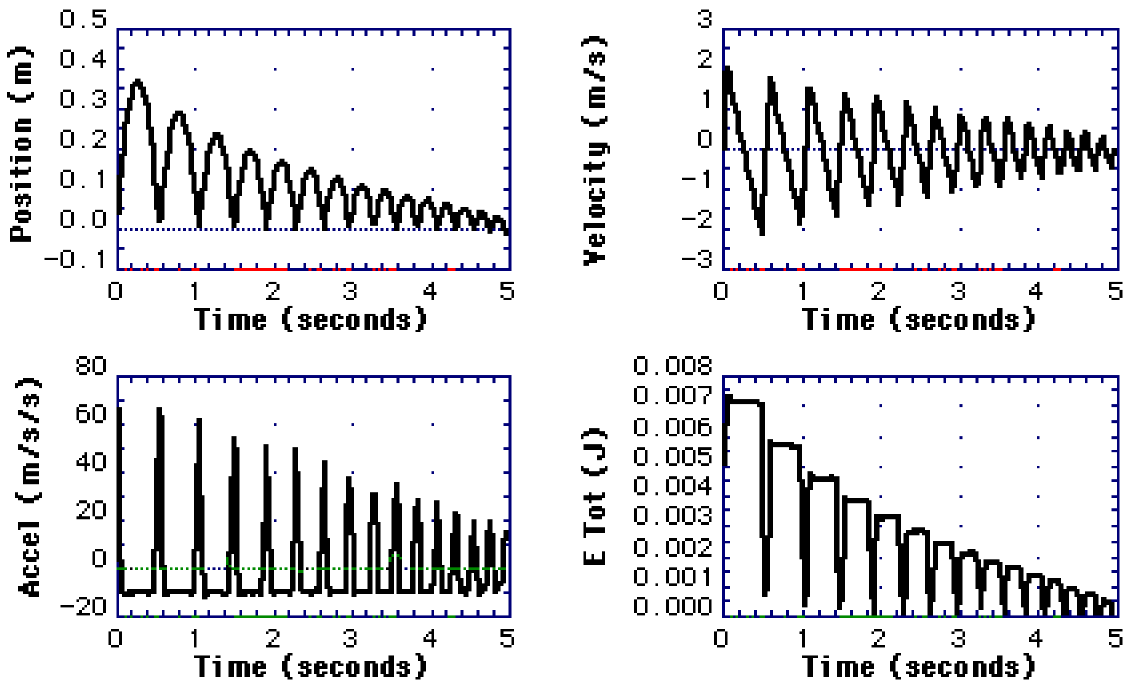

Figure 10

The motion is weakly damped by viscous forces: the acceleration differs only of 1% from g ($a \approx 9.7$ m/s^2). We may calculate also here the total energy E_{tot} as the sum of the potential energy $U=mgx$ and the kinetic energy $E_C=mv^2/2$, (the ball mass is m=2 grams).

We see that the energy is nearly constant during the upward-downward motion, and the energy loss takes place essentially during the collision, where the kinetic energy is converted into elastic energy (peaks towards E=0).

This elastic energy is partially given back as kinetic energy after the collision, and partially dissipated as sound (mechanical energy leaving the ball) and heat (increasing the temperature of the ball and the ground).

Conclusions

We suggest that by interfacing the Maxwell wheel with a PC we can transform a lab experience into a fruitful opportunity not only to strengthen students' knowledge of mechanics but also to emphasize the role of models in constructing physics knowledge. The data acquisition system can be a powerful cognitive tool by allowing a real time visualization of the relevant variables selected by the students according to any particular model they wish to test, and by encouraging the students to play the game of gradually refining the schematization in order to reach a satisfactory agreement with experimental data. Furthermore the opportunity offered by the system to easily compare descriptions of different phenomena and to detect common and uncommon features may help the students to better appreciate the potentialities of the kinematic description of motion.